# ASSESSMENT OF THE ACCURACY OF A LASER-TYPE DOPPLER FLOW METER WITH A GAUSSIAN DISTRIBUTION OF LIGHT BEAM INTENSITY

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16. Abstract

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# ASSESSMENT OF THE ACCURACY OF A LASER-TYPE DOPPLER FLOW METER WITH A GAUSSIAN DISTRIBUTION OF LIGHT BEAM INTENSITY

G. A. Barill and V. S. Sobolev /1981*

The problem of the measurement accuracy is very important when designing laser equipment for measuring the velocities of streams of liquids and gases. A rough estimate of the potential accuracy of a laser-type Doppler flow meter (LDFM) was made in [1] for a cosine approximation of the light intensity distribution in the region of the focal point. This article presents the results of studying the accuracy of the most promising LDFM with amplitude division of the laser beam and a gaussian distribution of the light intensity in the beams. It is assumed that the system for the signal electron processing is based upon the transformation of the "frequency-code" or "frequency-analog" principle and gives the average velocity in a very large time period and the "instantaneous" value in the form of the analog signal or codes in a small time interval.

A study is made of the methodical errors of the LDFM caused only by the random nature of the particle appearance in the focal region of the device. It is assumed that the optical portion of the LDFM is based on the system given in [2], in which additive noise of the laser and the low frequency component of the Doppler signal are compensated, and the electron portion

* Numbers in margin indicate pagination in original foreign text.

contains a very narrow-band slave filter. Due to this, the noise of the laser and the photoreceiver are not taken into account during the analysis.

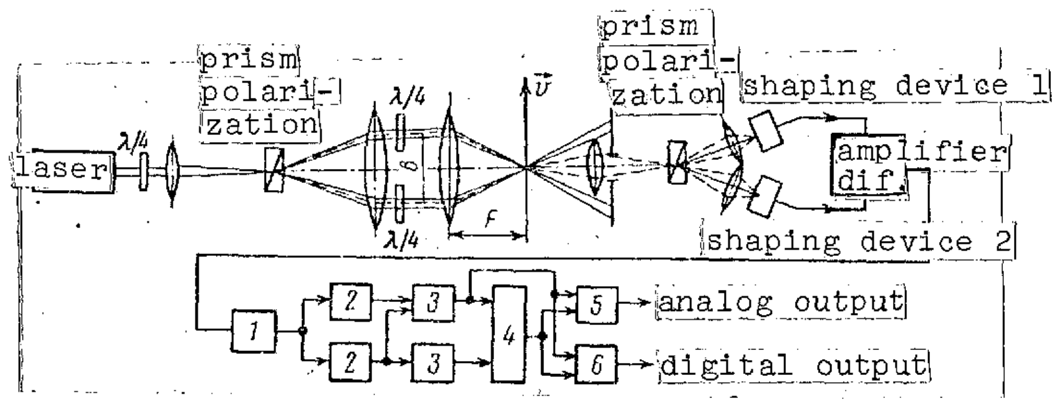
The following expression describes the Doppler signal at the photoreceiver output for the LDFM design with amplitude division of the laser ray and gaussian incident beams (Figure 1) /1982 (under the condition that there is compensation or filtration of the low-frequency component) [1]

$$i(t)=\sum_{i=1}^{N} A_i \exp[-(\gamma\omega_D(t-t_i))^2]\cos\omega_D(t-t_i), \tag{1}$$

where i(t) is the running value of the photoreceiver current; $A_i$ — amplitude of the envelope of the signal transmitted by the $i^{th}$ particle; $A_i$ depends on the laser power, the sensitivity of the photoreceiver, and the dimensions and optical properties of the particle; $t_i$ — moment at which the $i^{th}$ scattering particle enters the center of the ray interference field; $\omega_D$ — circular Doppler frequency; N — number of particles passing through the focal region during the observation period; $\gamma$ — parameter of the envelope of the Doppler signal from one particle; the value of $\gamma$ for gaussian beams is determined by the expression $\gamma=d/2\sqrt{2}b$, where d — diameter of the laser ray at the level at which its relative intensity is decreased by a factor of exp(—2); b — distance between the beams in the lens plane.

Analyzing the exponent of (1), we can readily see that the parameter $\gamma$ determines the length of the Doppler radio beam, obtained from one particle, and is thus related to the number of Doppler periods M, confined in the level exp(—2) of the envelope:

$$\gamma=\sqrt{2}/\pi M. \tag{2}$$




1- slave filter; 2- threshold amplifiers; 3- pulse shaping device; 4- strobe pulse shaping device; 5- frequency detector with memory; 6- frequency meter.

Thus M equals the number of the interference bands in the interference field.

The frequency, which is proportional to the velocity of motion, is an informative parameter of the Doppler signal. The random pile-up of signal pulses from each particle leads to fluctuations in the total signal phase and, consequently, to deviations in its frequency from the Doppler frequency even when the flow velocity (of the particle) is unchanged. These deviations lead to errors in determining the average flow velocity and cause noise equivalent to a certain turbulence, even when laminar flows are studied. A determination of this error and an assessment of the noise of the LDFM output — if by noise we mean the mean square deviations of the particle from its average values — is the purpose of this study.

It is known [3] that for normal stationary random processes, the values of the signal mean frequency and its mean square fluctuations are arbitrary autocorrelation functions of the process.

Assuming that the particle flow has a Poisson characteristic and taking into account the formula of Campbell [3], we may readily obtain the autocorrelation function of the Doppler signal, since in this case it coincides with the autocorrelation function of the signal from an individual particle. As is known, for an ergodic random process f(t), the normed autocorrelation function is determined by

$$R(\tau)=\frac{\lim\limits_{T\to\infty}\frac{1}{T}\int\limits_{-T}^{T} f(t)\,f(t+\tau)\,dt}{\lim\limits_{T\to\infty}\frac{1}{T}\int\limits_{-T}^{T} f^2(t)\,dt}. \qquad (3)$$

/1983

Substituting the expression for the signal from an individual particle from (1) into (3), and after the corresponding transformations, we obtain

$$R(\tau)=\exp\left[-\frac{(\gamma\omega_D\tau)^2}{2}\right]\cos\omega_D\tau. \qquad (4)$$

Let us now determine the average frequency of the Doppler signal and the mean square fluctuations of the particle or the equivalent noise. We will assume that the average frequency is the average number of signal outputs at the zero level per unit time. Assuming that the concentration of particles scattering light in the flow is large, and that the flow is laminar, we may assume that the Doppler signal is a stationary ergodic process with a gaussian distribution. It is known [3] (Formula 9.4.6) that for this type of process

$$N=\frac{1}{2\pi}\sqrt{-R''(0)}, \qquad (5)$$

where $R''(0)=d^2R(\tau)/d\tau^2$ when $\tau = 0$. Determining R" (0) from (4) and substituting the value found in (5), we obtain

$$N=\frac{\omega_D}{2\pi}\sqrt{1+\gamma^2}=f_D\sqrt{1+\gamma^2}. \qquad (6)$$

Taking the fact into account that in real LDFM designs, the parameter $\gamma < 0.1$, since the diameter of the laser ray is always much less than the distance between the divided beams, we may transform expression (6) to the following form

$$N \simeq f_D\left(1 + \frac{\gamma^2}{2}\right). \qquad (7)$$

If N— is the average frquency of the Doppler signal and $f_D$— is the true frequency corresponding to the laminar flow velocity, then the relative error in measuring the frequency will equal

$$\delta f = \frac{N - f_D}{f_D} = \frac{\gamma^2}{2} = \frac{1}{(\pi M)^2}. \qquad (8)$$

Thus, the error caused by applications of partial signals from individual particles is always positive and inversely proportional to the square of the number of interfering bands M in the interference field of the laser rays.

For the typical case when M = 20, we have $\delta f = 0{,}03\%$ .

Since the value of M is always constant for a given optical system, in calibrating the flow meter, a correction may be introduced which takes into account the error examined.

The particle velocity $v$ is related to the Doppler displacement of the frequency by the known relationship

$$v = \frac{\lambda f_D}{2\sin(\theta/2)},$$

where $\lambda$— is the length of the light wave emitted by the laser; $\theta$— angle between the light beams which are incident and scattered. Consequently, the total error of the velocity measurement is determined both by the accuracy of measuring the frequency $f_D$ and the accuracy of determining the angle $\theta$. In "Doppler frequency-code" transformation systems, the error of measuring the frequency comprises thousandths of a percent which, in order of magnitude, is less than the pile-up error. The error in determining the $\theta$ angle in existing LDFM systems is 0.1 - 0.2%. In the case of absolute measurements of the flow velocities, this error is decisive since it makes the greatest contribution to the resulting error. In the case of relative measurements, the pile-up error is decisive, which may be taken into account by introducing the correction.

We may use the fluctuations of the "instantaneous" frequency of the Doppler signal, caused by the pile-up of signals from individual particles, as the estimate of the error arising during measurements of the "instantaneous" velocity of turbulent flows. If we use the mean square deviation of the number of signal outputs at the zero level per unit time as the fluctuations of the frequency, we may make such an estimate. It is known that for the normal random process with an autocorrelation function of the form

$$R(\tau)=\rho(\tau)\cos\omega\tau$$

the dispersion $D_n$ of the number of signal transitions through /1984
zero during the measurement time $T$ ($T \gg \tau_0$, when $\tau_0$ — correlation time) is determined as follows [3] (Formula 1.7.23):

$$D_n = T\frac{2}{\pi^2}\int_0^T \frac{(\rho')^2}{1-\rho^2(\tau)}\,d\tau. \qquad (9)$$

Taking (4) into account, we assume that

$$\rho(\tau)=\exp\left[-\frac{(\gamma\omega_D\tau)^2}{2}\right].$$

Assuming that the measurement time T is much greater than the correlation time, we may determine the integral (9) in limits from zero to ∞. Then

$$D_n = T\frac{2}{\pi^2}\frac{1}{2}\gamma\omega_D\Gamma\left(\frac{3}{2}\right)\xi\left(\frac{3}{2},1\right) = \frac{2{,}612}{2\pi\sqrt{\pi}}\gamma\omega_D T, \tag{10}$$

where $\Gamma(3/2)$ — is the gamma function; $\xi(3/2,1)$ — zeta function of Riemann. The mean square deviation $\sigma_f$ of the number of positive zero-transitions of the signal per unit time is determined as

$$\sigma_f = \frac{\sigma_n}{2T} = \frac{\sqrt{D_n}}{2T} \approx \frac{0{,}41 f_D}{\sqrt{MS}}, \tag{11}$$

where $S = T/T_D = Tf_D$ — relative duration of measurement time; M— just as previously, the number of interference bands in the focal plane.

The signal/noise ratio may be determined as follows: [with allowance for (7)]

$$U_{s/n} = \frac{\sigma_f}{N} = \frac{0{,}41 f_D}{\sqrt{MS}\, f_D\left(1+\frac{\gamma^2}{2}\right)} \simeq \frac{0{,}41}{\sqrt{MS}}, \tag{12}$$

where N— average number of signal outputs at the zero level per unit time determined from (7).

Thus, the magnitude of the signal/noise ratio for laminar gradientless flows is determined exclusively by the relative width M of the signal pulse of a single particle and by the relative measurement time S and increases with a decrease in the indicated values. For typical M = 20 and S = 1000, the signal/noise ratio is 0.3%. However, if the time of a single measurement equals $10T_D$, i.e., S = 10, the relative noise increases to 3%.

If we assume a linear-piecewise approximation for the signal instantaneous frequency, then in accordance with the Kotel'nikov theorem, the pass band of the velocity pulsation

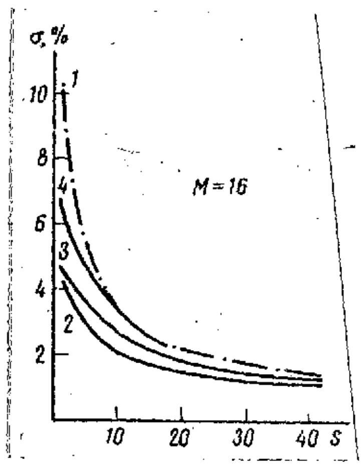



Figure 2.

frequencies is determined according to the formula

$$F = \frac{1}{2T} = \frac{f_D}{2S} \qquad (13)$$

For S = 1000 the maximum frequency of the pulsation spectrum will be 2,000 times smaller than the Doppler frequency. Taking the fact into account that the Doppler frequency may be selected very large, the relationship (13) satisfies the requirements of the majority of problems of hydrodynamics and aerodynamics.

As was indicated above, the results of the statistical analysis of the "frequency" of the Doppler signal are valid only in the case of a normal distribution of the latter. However, a real LDFM signal is far from always a normal random process. If the particle concentration is small, or if there is a very high spatial resolution of the equipment, the signal will have individual, or rarely intersecting, radio pulses.

To determine the methodical error in this case and the limits of applicability of the relationships obtained above, the Doppler signal was modeled on a computer. The program /1985
was formulated for a Poisson flow of particles. A curve was compiled for the distribution of the signal values, the length of each output at the zero level was determined, and the statistical characteristics were obtained for the values reached (converted to "frequency") for a given averaging time. Figure 2 gives the results of the modeling, showing the curves characterizing the signal/noise ratio at the LDFM output as a function of

the averaging time. Curve 1 was based on Expression (12), obtained from an analytical examination of a gaussian Doppler signal, and curves 2, 3, 4 were obtained by mathematical modeling for different concentrations of the scatterers. It may be seen from Figure 2 that curves 1 and 4, during an averaging time greater than $10T_D$, completely coincide, i.e., beginning at concentrations at which there are no less than four particles at the same time in the flow region examined, the characteristics of the real signal in which we are interested coincide with the characteristics of the corresponding gaussian process. To assess the LDFM errors, we may use Expressions (8) and (12). An analysis of the curves in Figure 2 shows that the smaller is the particle concentration, the smaller is the signal/noise ratio. However, it must not be forgotten that as the signal in this case we use the constant value which is equivalent to the average velocity of the particles (flow). If the velocity fluctuates, with a small concentration of particles, loss of information regarding the high frequency spectral region for these fluctuations becomes unavoidable.

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